Greedy Algorithms
CSCI 432

Single Room Scheduling

Goal: Assign as many courses as possible to a single classroom.

Single Room Scheduling

Input:

- $C = \{c_1, c_2, \dots, c_n\}$ – set of courses that need rooms.
- $c_i = [s_i, f_i)$ – start and finish times for each course.

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Goal: Select a maximum sized subset of compatible courses.

I.e., Fill a single room up with the most possible courses.

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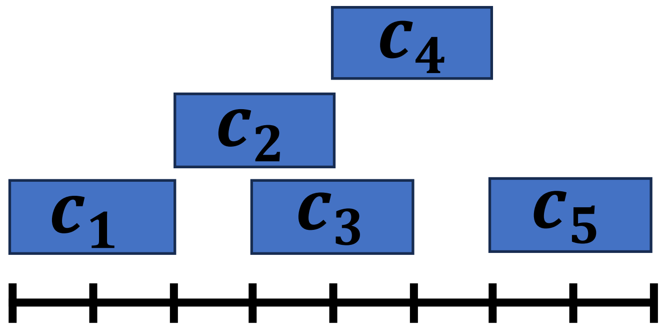
s_i	1	3	4	5	7
f_i	3	5	6	7	9



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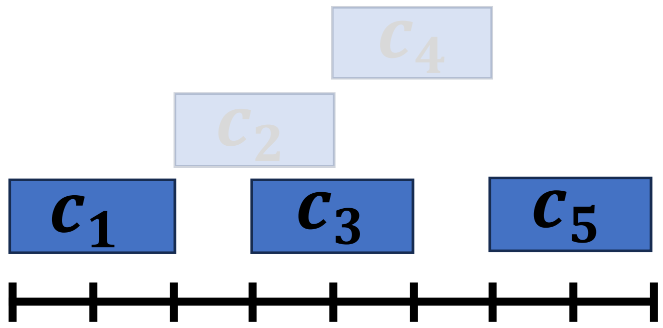

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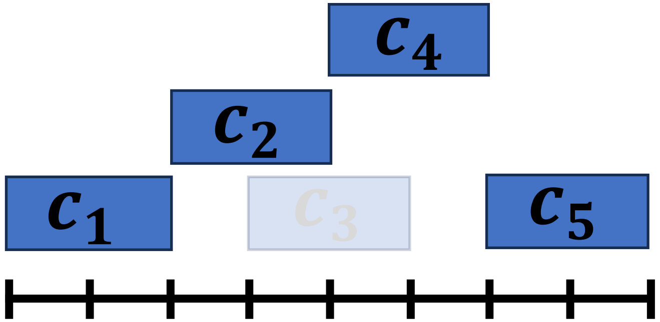

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Greedy selection criteria?

Earliest start time first.

In each iteration, pick the course whose start time is earliest.

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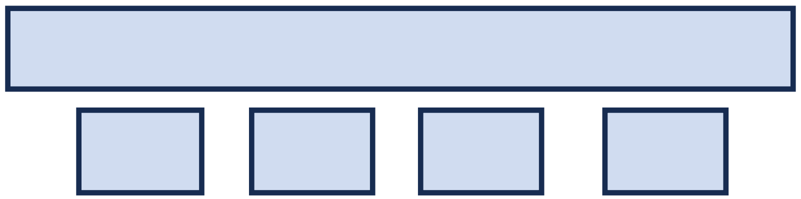

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Goal: Select a maximum sized subset of compatible courses.

Greedy selection criteria?

Smallest duration first.

In each iteration, pick the course that takes the least amount of time.

Single Room Scheduling

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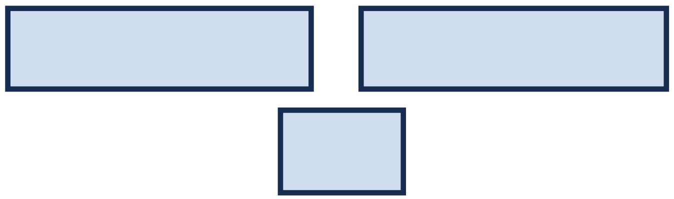

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Goal: Select a maximum sized subset of compatible courses.

Greedy selection criteria?

Smallest conflict first.

In each iteration, pick the course that overlaps with the smallest number of other courses.

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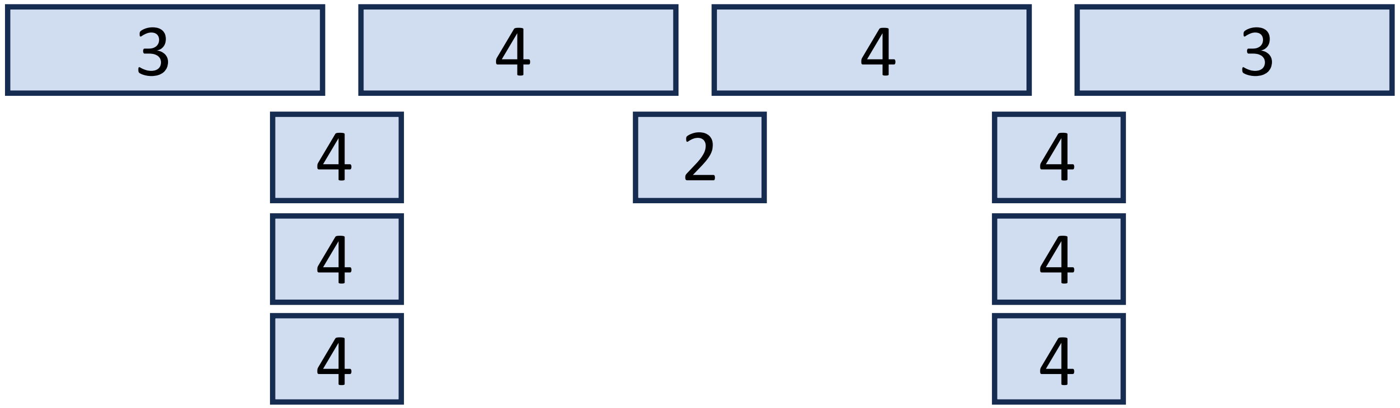

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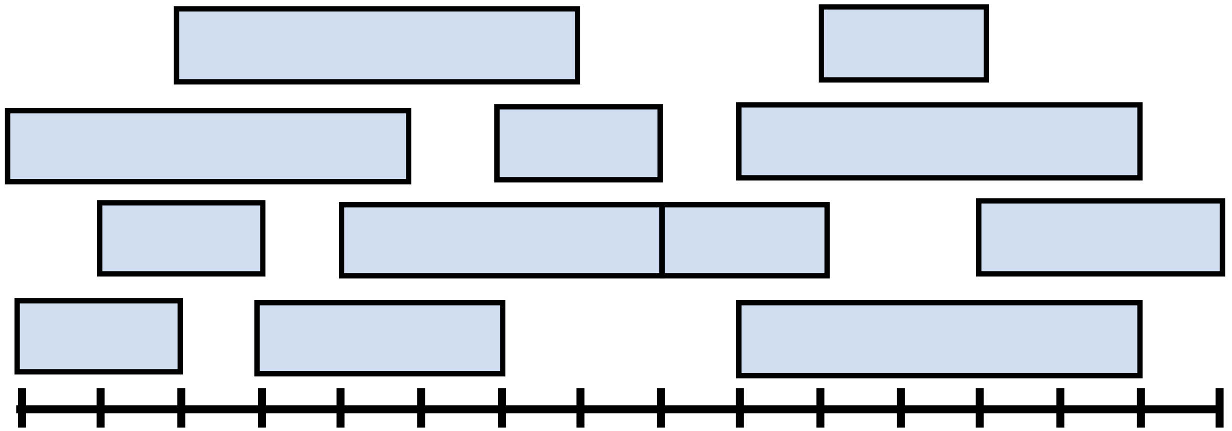

Greedy selection criteria?

Earliest compatible finish.

In each iteration, pick the course that ends earliest and is compatible with existing schedule.

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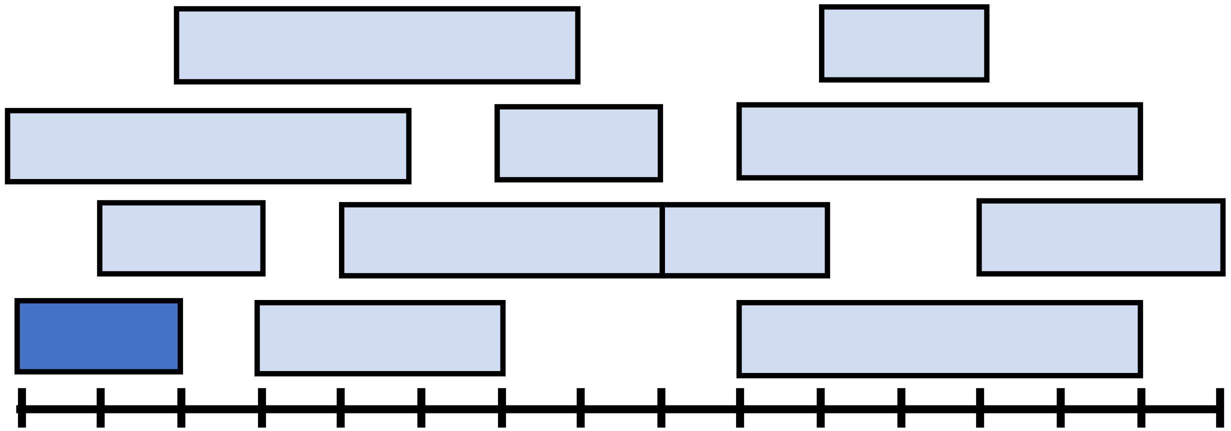

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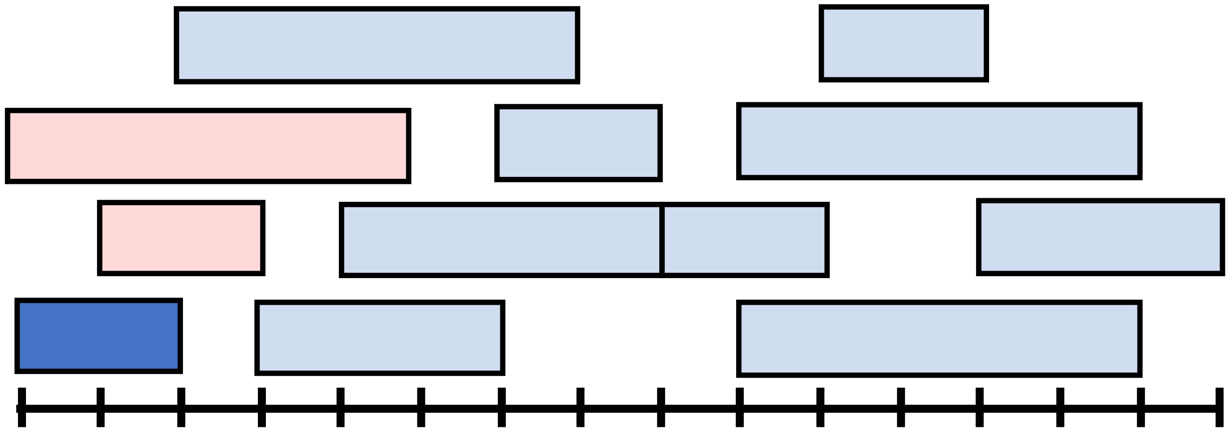

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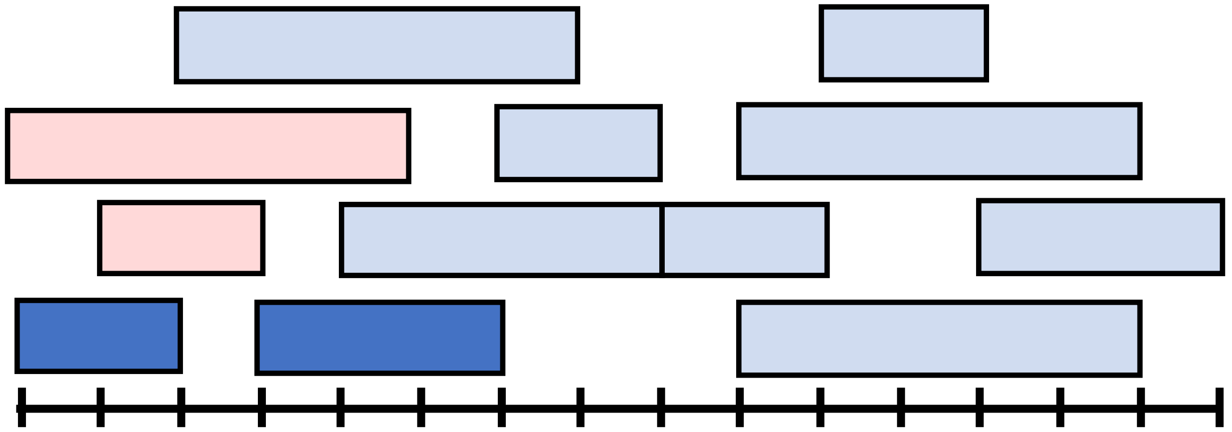

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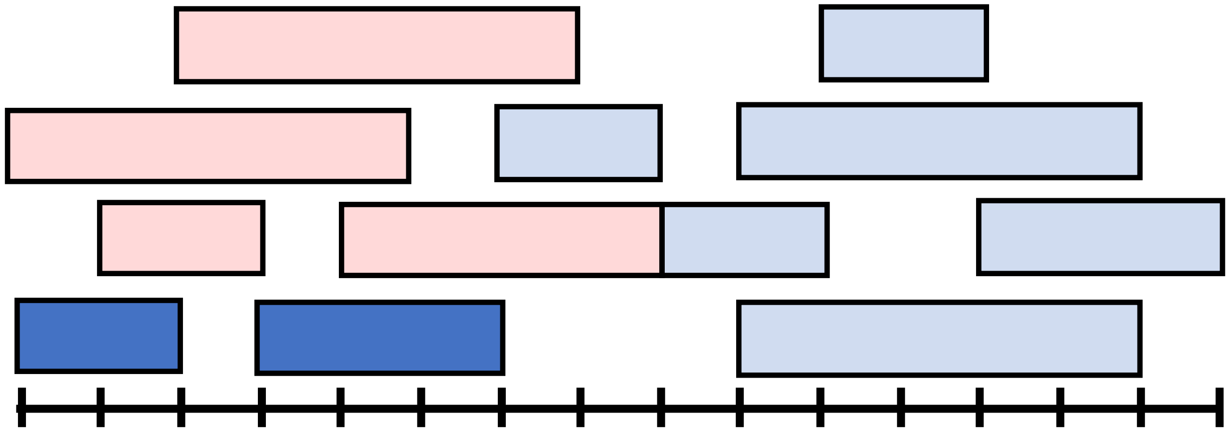

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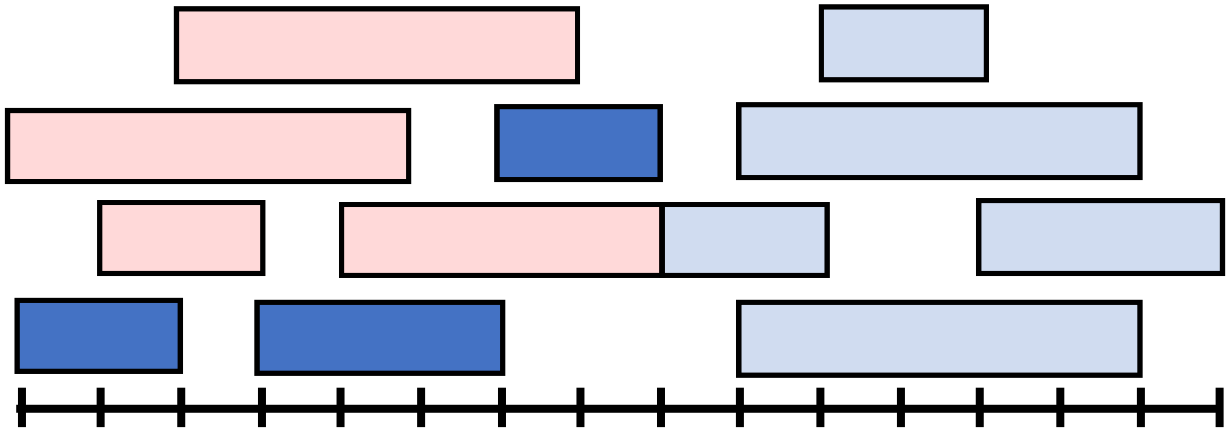

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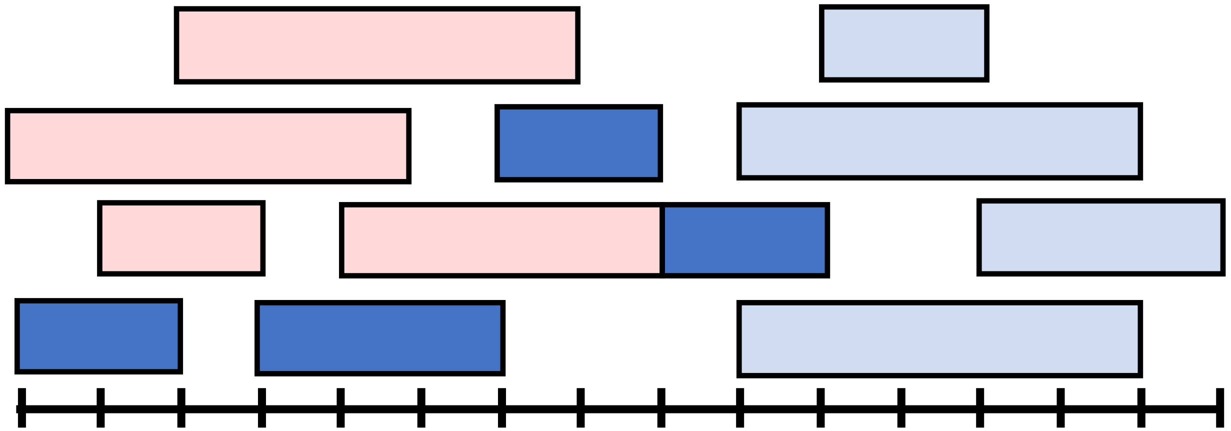

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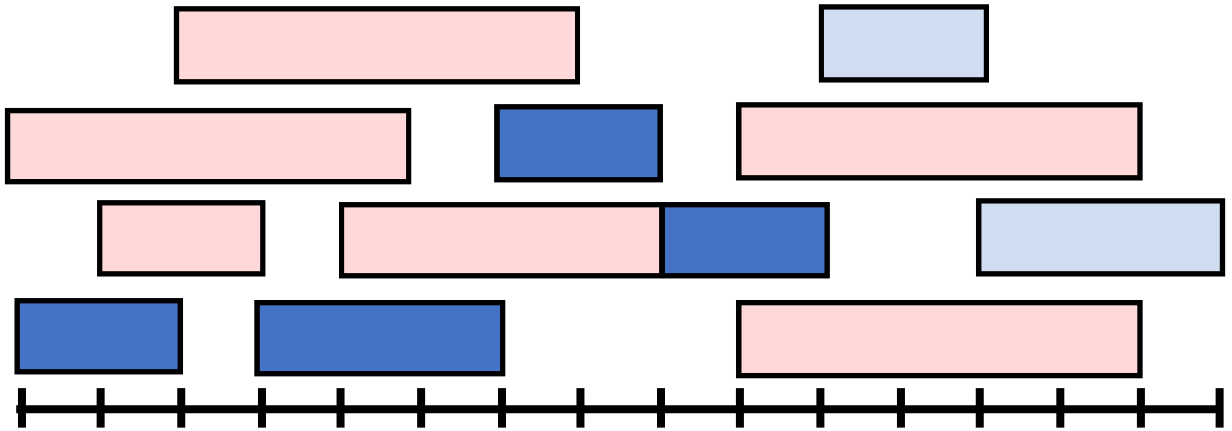

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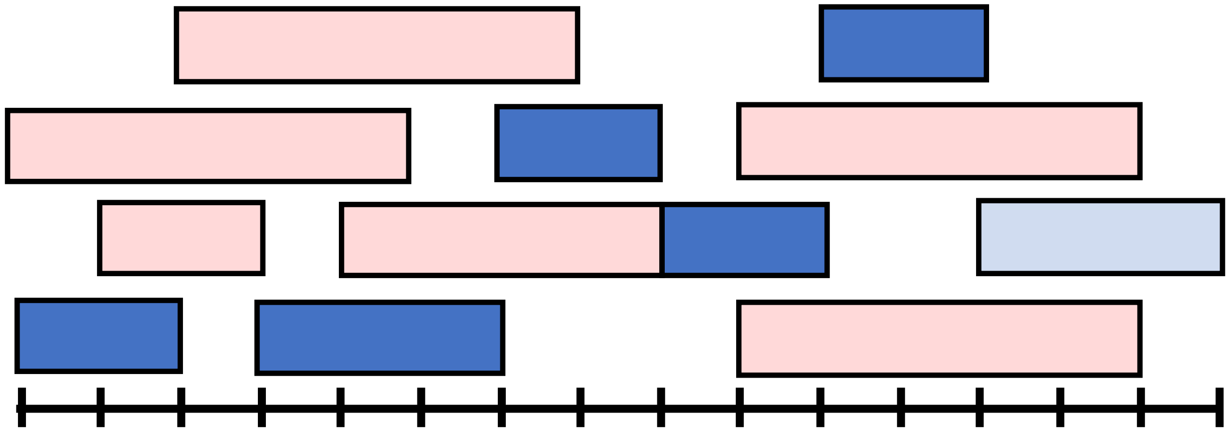

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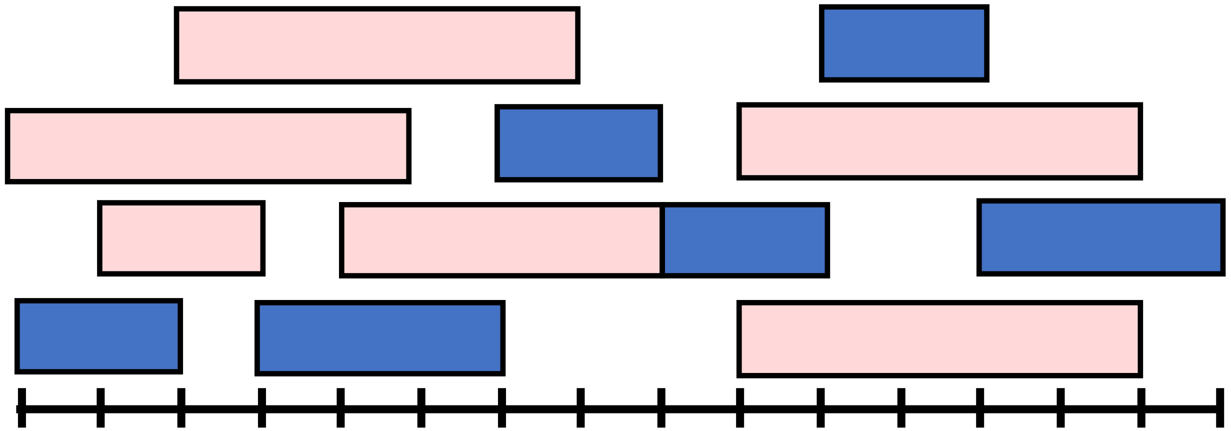

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Single Room Scheduling

Implementation Plan?

Single Room Scheduling

Implementation Plan?

1. Sort by increasing finish times.
2. Select first course.
3. Iterate through list looking for first compatible course.
4. Repeat.

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```
room_scheduling(courses C)
    C.sort_by_finish()
    selected = {C[1]}
    last_added = 1
    for i = 2 to C.length



    return selected
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Implementation Plan?

1. Sort by increasing finish times.
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```

Test to see if `C[i]` is compatible with courses already selected?

```
   return selected
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      if C[i].start ≥ C[last_added].finish
         selected.add(C[i])
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Running Time?

Validity?

Performance?

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Running Time? $O(n \log n)$

Validity?

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1. Sort by increasing finish times.
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Running Time? $O(n \log n)$

Validity? `selected` consists of compatible courses.

Performance?

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Implementation Plan?

1. Sort by increasing finish times.
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4. Repeat.

Running Time? $O(n \log n)$

Validity? `selected` consists of compatible courses.

Performance? Is `selected` the largest possible subset?

Single Room Scheduling

Greedy decision: Select the next course with the earliest compatible finish time.

Proof of optimality:

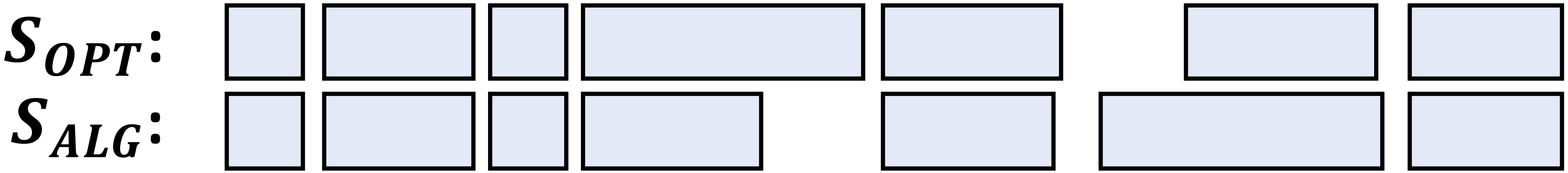
Plan: Turn a hypothetical optimal solution into the algorithm’s solution without changing the cost (i.e., number of courses) and without violating course compatibility.

Single Room Scheduling

<u>Greedy decision:</u> Select the next course with the earliest compatible finish time.

<u>Proof of optimality:</u> Let C be the set of courses, $S_{ALG} \subseteq C$ be the greedy algorithm's selection, and $S_{OPT} \subseteq C$ be an optimal selection, all sorted by increasing finish time.

<u>Plan:</u> Turn a hypothetical optimal solution into the algorithm's solution without changing the cost (i.e., number of courses) and without violating course compatibility.



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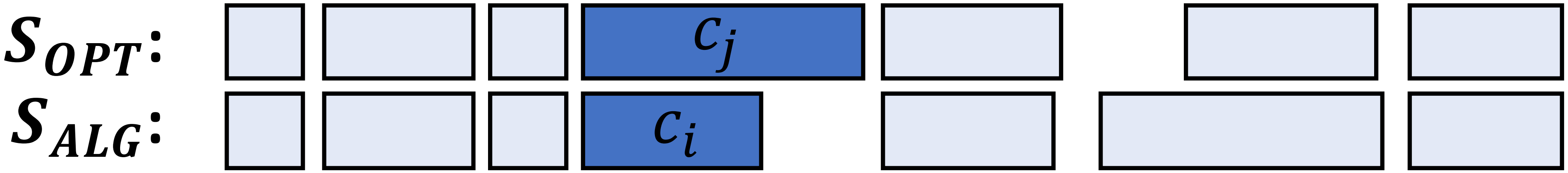
Suppose $S_{ALG}[i] = S_{OPT}[i]$, for all $i < k$ and $S_{ALG}[k] = c_i \neq c_j = S_{OPT}[k]$.

Suppose S_{ALG} and S_{OPT} schedule the same courses up until course k.

<u>Plan:</u> Turn a hypothetical optimal solution into the algorithm's solution without changing the cost (i.e., number of courses) and without violating course compatibility.



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Create the revised schedule $S'_{OPT} = S_{OPT} \setminus \{c_j\} \cup \{c_i\}$. (I.e., Swap $S_{ALG}[k]$ for $S_{OPT}[k]$)

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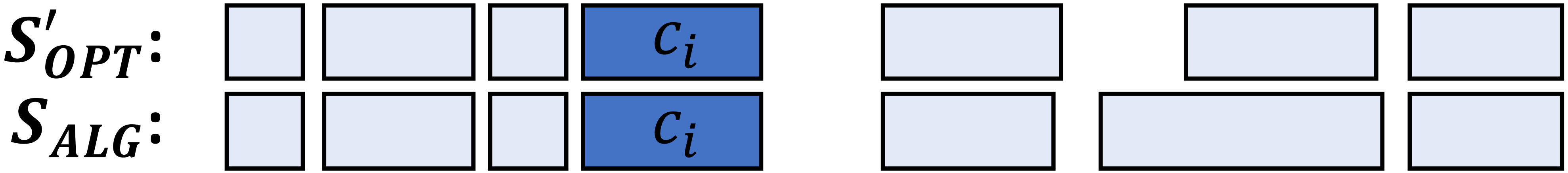

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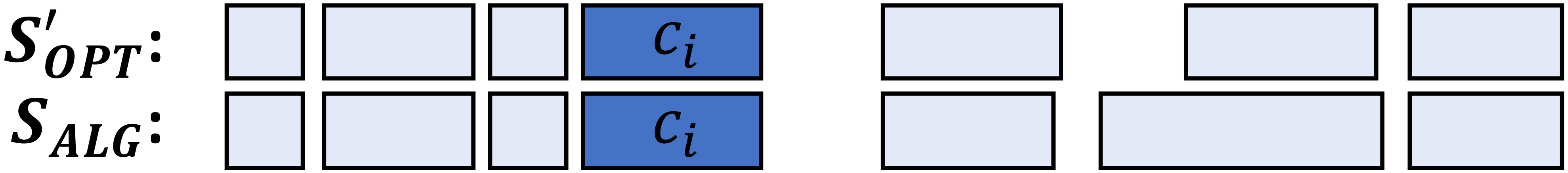
Will S'_{OPT} be valid?



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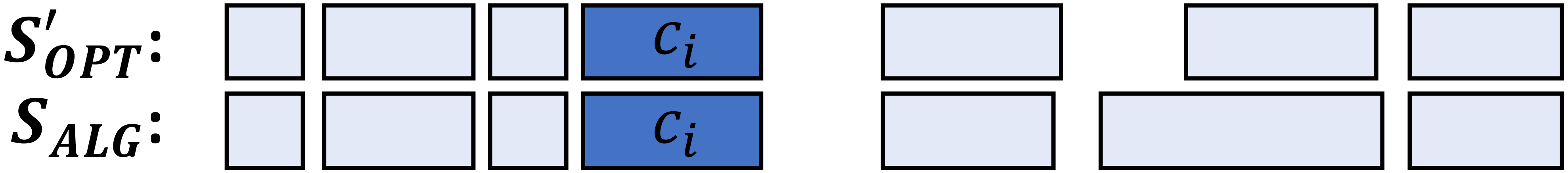
Will S'_{OPT} be valid?

Need to check and see if c_j

messed up any compatibilities.



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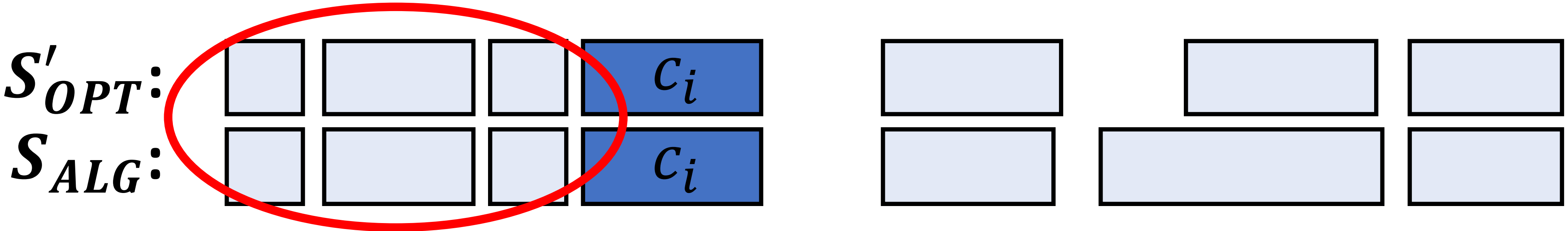
Create the revised schedule $S'_{OPT} = S_{OPT} \setminus \{c_j\} \cup \{c_i\}$. (I.e., Swap $S_{ALG}[k]$ for $S_{OPT}[k]$)

c_i is compatible with previous courses in S'_{OPT} since $S_{ALG}[i] = S_{OPT}[i] = S'_{OPT}[i]$, for all $i < k$

c_i is compatible with S_{ALG}, and S_{OPT} and S'_{OPT} share the same courses before c_i.



Single Room Scheduling

<u>Greedy decision:</u> Select the next course with the earliest compatible finish time.

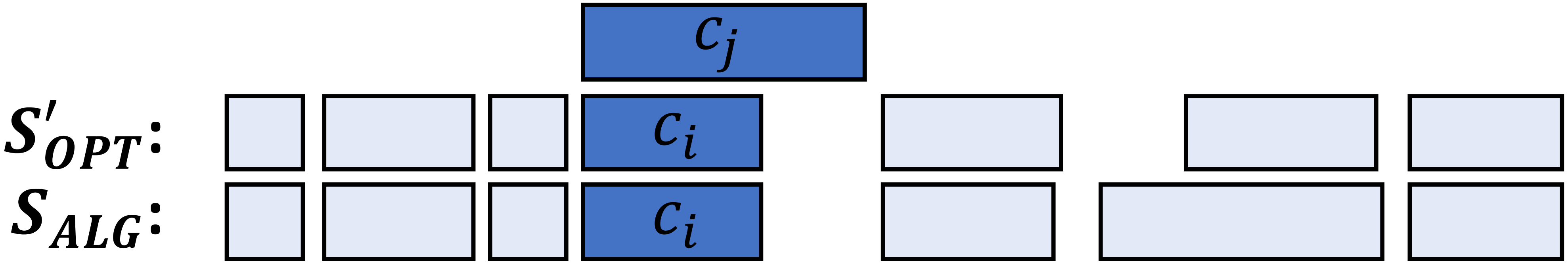
<u>Proof of optimality:</u> Let C be the set of courses, $S_{ALG} \subseteq C$ be the greedy algorithm's selection, and $S_{OPT} \subseteq C$ be an optimal selection, all sorted by increasing finish time.

Suppose $S_{ALG}[i] = S_{OPT}[i]$, for all $i < k$ and $S_{ALG}[k] = c_i \neq c_j = S_{OPT}[k]$.

Create the revised schedule $S'_{OPT} = S_{OPT} \setminus \{c_j\} \cup \{c_i\}$. (I.e., Swap $S_{ALG}[k]$ for $S_{OPT}[k]$)

c_i is compatible with previous courses in S'_{OPT} since $S_{ALG}[i] = S_{OPT}[i] = S'_{OPT}[i]$, for all $i < k$

c_i is compatible with subsequent courses in S'_{OPT} since $f_i \leq f_j$. Otherwise, the greedy algorithm would have selected c_j instead of c_i.



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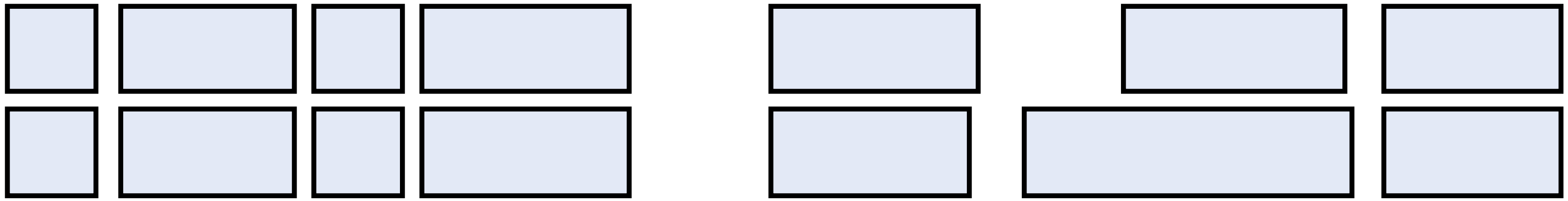
So S'_{OPT} is a valid schedule with the same number of courses as S_{OPT}, so S'_{OPT} is also optimal.

$\boldsymbol{S'_{OPT}}$:
$\boldsymbol{S_{ALG}}$:



Single Room Scheduling

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We can then proceed inductively and show that each course in S_{OPT} can be replaced by the corresponding course in S_{ALG} without violating compatibility.

Single Room Scheduling

<u>Greedy decision:</u> Select the next course with the earliest compatible finish time.

<u>Proof of optimality:</u> Let C be the set of courses, $S_{ALG} \subseteq C$ be the greedy algorithm's selection, and $S_{OPT} \subseteq C$ be an optimal selection, all sorted by increasing finish time.

Suppose $S_{ALG}[i] = S_{OPT}[i]$, for all $i < k$ and $S_{ALG}[k] = c_i \neq c_j = S_{OPT}[k]$.

Create the revised schedule $S'_{OPT} = S_{OPT} \setminus \{c_j\} \cup \{c_i\}$. (I.e., Swap $S_{ALG}[k]$ for $S_{OPT}[k]$)

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c_i is compatible with subsequent courses in S'_{OPT} since $f_i \leq f_j$. Otherwise, the greedy algorithm would have selected c_j instead of c_i.

So S'_{OPT} is a valid schedule with the same number of courses as S_{OPT}, so S'_{OPT} is also optimal.

We can then proceed inductively and show that each course in S_{OPT} can be replaced by the corresponding course in S_{ALG} without violating compatibility. Since replacing every course in S_{OPT} with the courses in S_{ALG} keeps the solution optimal, S_{ALG} must be optimal. (i.e., we translated S_{OPT} into S_{ALG} at no extra cost).

Single Room Scheduling

```
room_scheduling(courses C)
   C.sort_by_finish()
   selected = {C[1]}
   last_added = 1
   for i = 2 to C.length
      if C[i].start ≥ C[last_added].finish
         selected.add(C[i])
         last_added = i
   return selected
```

Implementation Plan?

1. Sort by increasing finish times.
2. Select first course.
3. Iterate through list looking for first compatible course.
4. Repeat.

Running Time? $O(n \log n)$

Validity? `selected` consists of compatible courses.

Performance? `selected` is the largest valid subset.